

Linear Analysis of Shock Turbulence Interaction

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Outline

Linear Interaction Analysis (LIA)

- Problem formulation
- Upstream waveforms
- Governing equations and Boundary conditions
- Downstream solution

Problem formulation

- Acoustic mode
- Vorticity mode
- Entropy mode



TURBULENT FLOW

- | | | |
|---------------|---|----------------------|
| • Pressure | → | Acoustic |
| • Density | → | Acoustic , Entropy |
| • Temperature | → | Acoustic , Entropy |
| • Velocity | → | Vorticity , Acoustic |

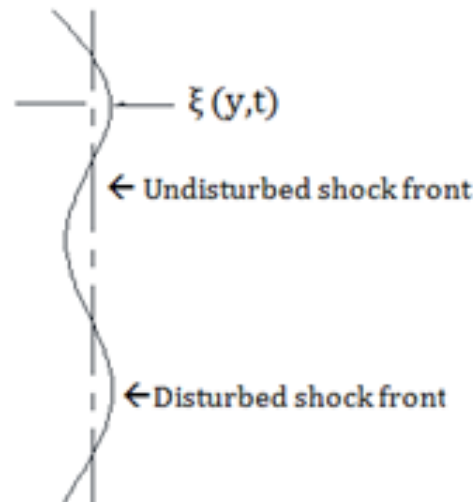
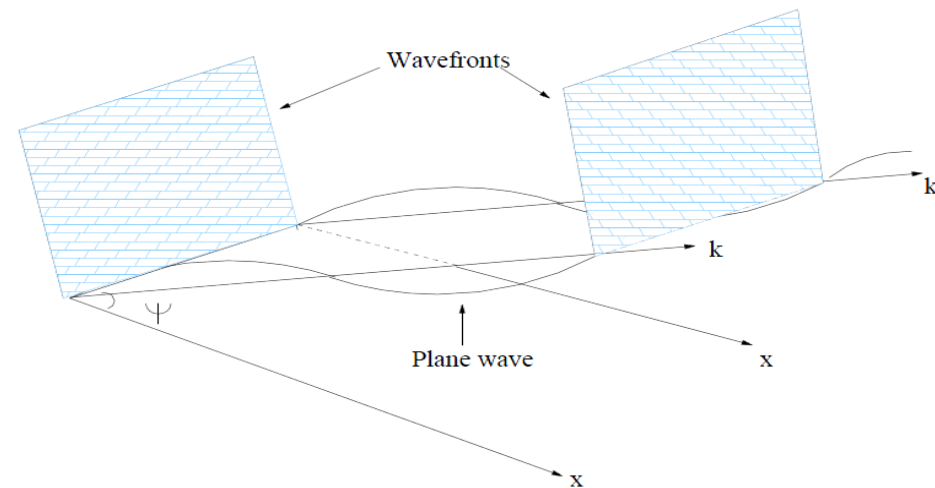
Linear Interaction Analysis

- Upstream – Steady, Uniform 1D mean flow
- Isotropic Turbulence
- Small fluctuations

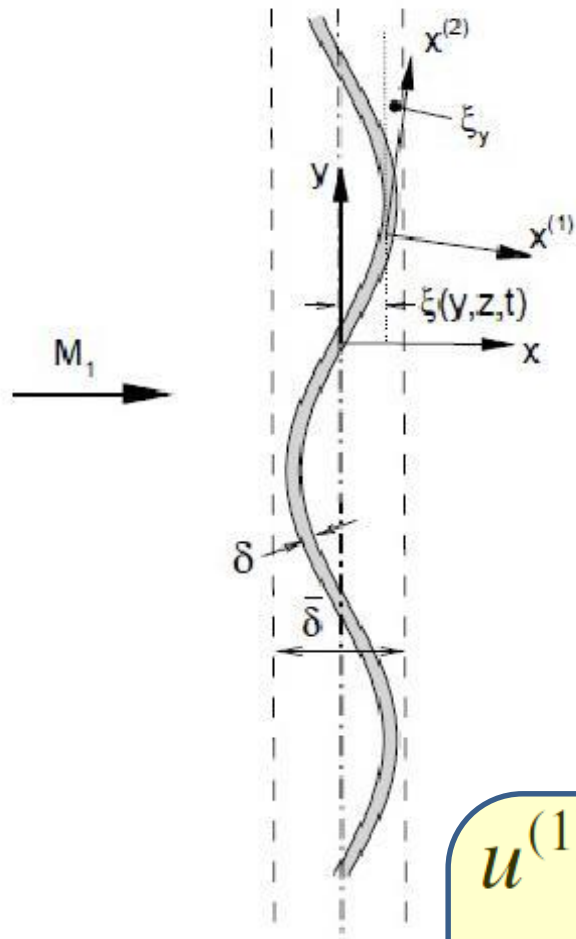


$$\frac{u'}{\overline{U}}, \frac{v'}{\overline{U}} \ll 1$$

- Shock distortion



Problem formulation



$$x^{(1)} = x - \xi - y\xi_y - z\xi_z$$

$$x^{(2)} = x\xi_y + y$$

$$x^{(3)} = x\xi_z + z$$

$$u^{(1)} = u - \xi_t - v\xi_y - w\xi_z \simeq \bar{u} + u' - \xi_t$$

$$u^{(2)} = u\xi_y + v \simeq \bar{u}\xi_y + v'$$

$$u^{(3)} = u\xi_z + w \simeq \bar{u}\xi_z + w'$$

LIA-Upstream Waveforms

$$\begin{aligned}\frac{u_1'}{\bar{U}_1} &= l A_v e^{ik(mx+ly-U_1 mt)} \\ \frac{v_1'}{\bar{U}_1} &= -m A_v e^{ik(mx+ly-U_1 mt)} \\ \frac{\rho_1'}{\bar{\rho}_1} &= A_e e^{ik(mx+ly-U_1 mt)} \\ \frac{T_1'}{\bar{T}_1} &= -\frac{\rho_1'}{\bar{\rho}_1} \\ p_1' &= 0\end{aligned}$$

Linearized Rankine -
Hugoniot relations

+

Linearized Euler
Equations

Downstream
Waveforms

Boundary conditions at $x=0$ (Lin. R-H Relations)

$$\rho_1 u_1^{(1)} = \rho_2 u_2^{(1)}$$

$$p_1 + \rho_1 (u_1^{(1)})^2 = p_2 + \rho_2 (u_2^{(1)})^2$$

$$u_1^{(2)} = u_2^{(2)}$$

$$h_1 + \frac{(u_1^{(1)})^2}{2} = h_2 + \frac{(u_2^{(1)})^2}{2}$$

Linearize

$$\frac{u_2' - \xi_t}{\bar{U}_1} = B_1 \left(\frac{u_1' - \xi_t}{\bar{U}_1} \right) + B_2 \left(\frac{T_1'}{\bar{T}_1} \right)$$

$$\frac{\rho_2'}{\bar{\rho}_2} = C_1 \left(\frac{u_1' - \xi_t}{\bar{U}_1} \right) + C_2 \left(\frac{T_1'}{\bar{T}_1} \right)$$

$$\frac{p_2'}{\bar{p}_2} = D_1 \left(\frac{u_1' - \xi_t}{\bar{U}_1} \right) + D_2 \left(\frac{T_1'}{\bar{T}_1} \right)$$

$$\frac{v_2'}{\bar{U}_1} = \frac{v_1'}{\bar{U}_1} + E_1 \xi_y$$

LIA-Downstream waveforms

- Vorticity , Entropy fluctuations

$$e^{ik(mrx+ly-m\bar{U}_1t)}$$

- Acoustic fluctuations

$$p' = F(x) e^{ik(ly-m\bar{U}_1t)}$$

$$0 < \psi_1 < \psi_c$$

$$\psi_1 > \psi_c$$

$$F \sim e^{i\tilde{k}x}$$

(Propagating Regime)

$$F \sim e^{-\tilde{k}_i x} e^{i\tilde{k}_r x}$$

(Decaying Regime)

$$\cot^2 \psi_c = \left(\frac{a_2^2}{\bar{U}_1^2} - \frac{\bar{U}_2^2}{\bar{U}_1^2} \right)$$

$$\tilde{k} = \frac{-2km \frac{\bar{U}_2}{\bar{U}_1} + 2kl \frac{a_2}{\bar{U}_1} \sqrt{\frac{m^2}{l^2} - \left(\frac{a_2^2}{\bar{U}_1^2} - \frac{\bar{U}_2^2}{\bar{U}_1^2} \right)}}{2 \left(\frac{a_2^2}{\bar{U}_1^2} - \frac{\bar{U}_2^2}{\bar{U}_1^2} \right)}$$

Governing equations for $x \geq 0$

(Lin. Euler Eqns)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v) = 0$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho u v) = -\frac{\partial p}{\partial x}$$

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v u) + \frac{\partial}{\partial y}(\rho v v) = -\frac{\partial p}{\partial y}$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}(Eu) + \frac{\partial}{\partial y}(Ev) + \frac{\partial}{\partial x}(pu) + \frac{\partial}{\partial y}(pv) = 0$$

Linearize

$$\rho'_t + U \rho'_x = -\bar{\rho}(u'_x + v'_y)$$

$$u'_t + U u'_x = -\frac{1}{\bar{\rho}} p'_x$$

$$v'_t + U v'_x = -\frac{1}{\bar{\rho}} p'_y$$

$$s'_t + U s'_x = 0$$

+

$$\frac{p'}{\bar{P}} = \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}}$$

$$\frac{s'}{C_p} = \frac{p'}{\gamma \bar{P}} - \frac{\rho'}{\bar{\rho}}$$

Downstream waveforms

- Pressure $\longrightarrow$ Acoustic

$$\frac{p_2'}{\bar{P}_2} = K e^{i\tilde{k}x} e^{ik(l y - m \bar{U}_1 t)}$$

- Density, Temperature $\longrightarrow$ Acoustic, Entropy

$$\frac{\rho_2'}{\bar{\rho}_2} = \frac{K}{\gamma} e^{i\tilde{k}x} e^{ik(l y - m \bar{U}_1 t)} + Q e^{ik(m r x + l y - m \bar{U}_1 t)}$$

$$\frac{T_2'}{\bar{T}_2} = \frac{(\gamma - 1)}{\gamma} K e^{i\tilde{k}x} e^{ik(l y - m \bar{U}_1 t)} - Q e^{ik(m r x + l y - m \bar{U}_1 t)}$$

Downstream waveforms

- Velocity $\longrightarrow$ Vorticity, Acoustic

$$\frac{u_2'}{\bar{U}_1} = F e^{i\tilde{k}x} e^{ik(l y - m \bar{U}_1 t)} + G e^{ik(m r x + l y - m \bar{U}_1 t)}$$

$$\frac{v_2'}{\bar{U}_1} = H e^{i\tilde{k}x} e^{ik(l y - m \bar{U}_1 t)} + I e^{ik(m r x + l y - m \bar{U}_1 t)}$$

- Shock front

$$\frac{\xi_t}{\bar{U}_1} = L e^{ik(l y - m \bar{U}_1 t)}$$

$$\xi_y = -\frac{l}{m} L e^{ik(l y - m \bar{U}_1 t)}$$

Solution

Downstream Waveforms

$$\frac{u'_2 - \xi_t}{\bar{U}_1} = B_1 \left(\frac{u'_1 - \xi_t}{\bar{U}_1} \right) + B_2 \left(\frac{T'_1}{\bar{T}_1} \right)$$

$$\frac{\rho'_2}{\bar{\rho}_2} = C_1 \left(\frac{u'_1 - \xi_t}{\bar{U}_1} \right) + C_2 \left(\frac{T'_1}{\bar{T}_1} \right)$$

$$\frac{p'_2}{\bar{p}_2} = D_1 \left(\frac{u'_1 - \xi_t}{\bar{U}_1} \right) + D_2 \left(\frac{T'_1}{\bar{T}_1} \right)$$

$$\frac{v'_2}{\bar{U}_1} = \frac{v'_1}{\bar{U}_1} + E_1 \xi_y$$

$$\tilde{F} = \alpha \tilde{K}$$

$$\tilde{H} = \beta \tilde{K}$$

$$\tilde{I} = -\frac{mr}{l} \tilde{G}$$

$$\tilde{F} + \tilde{G} - \tilde{L} = B_1(l - \tilde{L}) - B_2 \frac{A_e}{A_v}$$

$$\frac{\tilde{K}}{\gamma} + \tilde{Q} = C_1(l - \tilde{L}) - C_2 \frac{A_e}{A_v}$$

$$\tilde{K} = D_1(l - \tilde{L}) - D_2 \frac{A_e}{A_v}$$

$$\tilde{H} + \tilde{I} = -m - E_1 \frac{l}{m} \tilde{L}$$

Thank you!